Optimization: Area of a Triangle

Lesson Information
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Subject:
Pre-Calculus Algebra
Topic:
Functions Graphing Geometry and Measurement
Technology:
Geometry Software
Level:
Difficult
Activity Structure:
Self-guided Problem Solving
Duration of Activity:
Whole Class Period Multiple Classes

Overview:

This lesson walks students through a classic optimization problem involving building the maximum area of a triangle, expressed in terms of an angle, θ. The lesson uses a worksheet in The Geometer's Sketchpad.

Learning Objectives:

- Students will be able to look for patterns and determine the largest area for a triangle.
- Students will be able to use this knowledge to solve similar problems
- Students will gain an understanding of the problem solving process used to solve optimization type problems
- To use trigonometry to express the area of a triangle in terms of an angle.

Materials:

- The Geometer's Sketchpad Software
- GSP Worksheet entitled "Trig"
- Paper
- Pencil
- Worksheet

Problem:

The length of each of the two equal sides of an isosceles triangle is 5 cm. The angle between the equal sides is θ. Find the value of θ such that the triangle's area is a maximum.
Note: use radians.

Exploration:

1. Drag point A to change the vertex angle of the triangle. Observe the changes in the base and height of the triangle as the vertex angle changes.

   a) What happens to the base of the triangle as the vertex angle changes?

   b) What happens to the height of the triangle as the vertex angle changes?

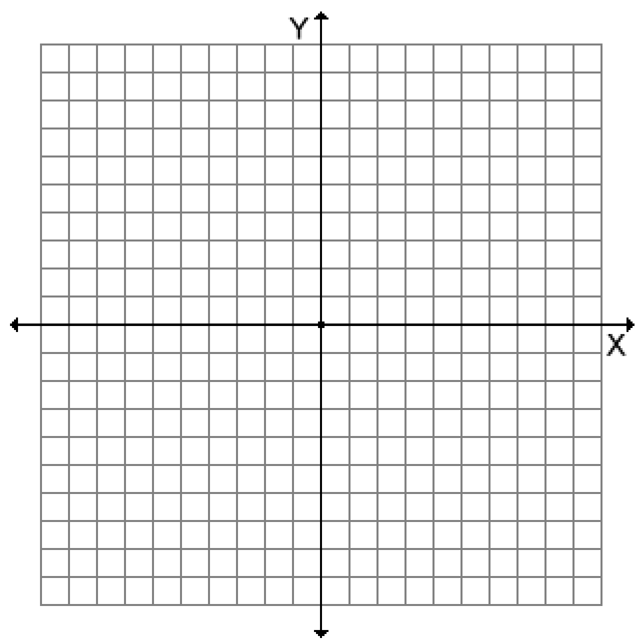
2. What vertex angle (θ) do you think will create the triangle with the maximum area?

3. Click on "Show Table." This table shows the base, height, and area of the triangle for a particular measure of θ. Change the vertex angle. How do the numbers in the table change, relative to one another?

4. Create a table for five different values of θ, including the one you proposed in question two. Copy your table below.

5. Is the triangle you proposed in question two the triangle with the largest area on your table? What does this tell you about the desired triangle?

6. Plot a rough sketch of area vs. θ on the coordinate axes below.



7. What type of mathematical relationship does this resemble?

8. Click on “Hide table” and then click on “Show graph.” Change the dimensions of the triangle. Watch the trace of the plotted point as the dimensions of the triangle change. What type of mathematical relationship does this resemble? Does this match your answer from question seven?

9. What point on the graph represents the maximum area of the triangle?

10. Drag point A to create the triangle you guessed in question two. Is the plotted point at the maximum value of your graph?

11. Click on “Move to maximum area.” How far was your hypothesized point from the desired point (in terms of the independent variable θ)?

12. Describe the shape of the triangle with the maximum area.

Generalization:

1. Consider an isosceles triangle with legs x and vertex angle θ. Draw a sketch of the triangle.

2. Write an expression for the area of the triangle in terms of x and θ. You may need to recall your double angle formulas in order to simplify your expression.

3. Because x is a parameter, it is possible to consider the area a function of θ. What type of function is this?

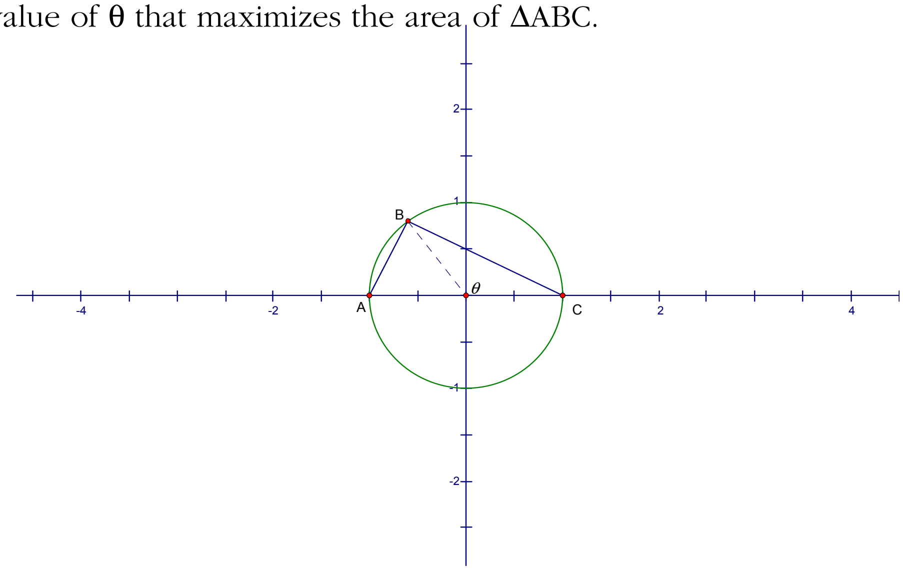
4. What is the domain of this function?

5. Were your predictions in questions 7 and 8 of the exploration correct? If not, explain what caused you to come to an incorrect conclusion.

6. Describe a method for finding the maximum value for this type of function.

Extensions:

1. Find the value of θ that maximizes the area of ΔABC.



2. A circular sector has radius r and central angle θ. The perimeter of the sector is 12. Find the values of r and θ that will create the sector with the maximum area.

3. The figure below shows a right circular cone with a slant height of 1 meter. Find the value of θ so that the cone's volume is a maximum.